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Russian Translation

On the Use of Artificial Satellites of the Earth to Check the General Theory of Relativity

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It can be stated at the present time that experimental verification has lead to convincing corroboration of the general theory of relativity (for example, see [1]). However, further verification of the theory is not unnecessary. In this connection, it is appropriate to indicate the possibility disclosed for the experimental verification of the general theory of relativity by using artificial satellites of the earth.

The method of radio radiation from the earth to the satellite could be used to detect the gravitational displacement of the frequency in the terrestrial field [2]. Here, the relative gravitational variation of the frequency ν equals:

$$(1) \qquad \frac{\Delta\nu}{\nu} = \frac{\varkappa M_{\eth}}{c^2}\left(\frac{1}{r_{\eth}} - \frac{1}{r_{\eth} + h}\right) \simeq \frac{gh}{c^2}\left(1 - \frac{h}{r_{\eth}}\right) = 1.09\cdot 10^{-18} h\left(1 - \frac{h}{r_{\eth}}\right)$$

where $\varkappa = 6.670\cdot 10^{-8}$ is the gravitational constant, $M_{\eth} = 5.98\cdot 10^{27}$ is the mass of the earth, $r_{\eth} = 6.36\cdot 10^{8}$ is the radius of the earth, $g = 981$ is the acceleration of gravity, h is the satellite height above the surface of the earth. When $h = 800$ km, then $\frac{\Delta\nu}{\nu} = 7.6\cdot 10^{-11}$, if $h \gg r_{\eth}$, then $\frac{\Delta\nu}{\nu} = 7\cdot 10^{-10}$ (the considered displacement in the terrestrial field is, understandably, violet and not red). At the same time, in principle, an accuracy of $\frac{\Delta\nu}{\nu} \gtrsim 10^{-12}$ could be attained by using an atomic clock (see [3]) and an accuracy of $\gtrsim 10^{-10}$ has already been attained [4]. Formula (1) is not applicable for the complete variation of the frequency of the satellite radiation since the Doppler frequency displacement must also be taken into account. It is easy to show, for example, that by using the general expression for the Doppler effect in a gravitational field (see [5], § 116), the gravitational and Doppler frequency shifts simply combine, in the $\sim \frac{v^2}{c^2}$ accuracy of interest, where the usual formula must be used for the Doppler shift, thus:

$$\left(\frac{\Delta\nu}{\nu}\right)_d = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}\cos\theta} \approx 1 + \frac{v}{c}\cos\theta - \frac{v^2}{2c^2}(1 - 2\cos^2\theta)$$

where v is the satellite velocity relative to the earth's surface and θ is the angle between the velocity and the line-of-sight. Hence, the instantaneous values must be taken for v and θ and the source acceleration never enters into the result; the corrections for satellites close to the earth, which are related to the earth's rotation (we speak of a measuring system coupled to the earth), are small and can be discarded (these corrections are $\frac{v_o^2}{c^2} \lesssim 2\cdot10^{-12}$, where $v_o = 4.6\cdot10^4$ is the speed of the earth's surface at the equator). The linear Doppler effect drops out when the satellite is observed at a $\theta = \frac{\pi}{2}$ angle but the quadratic effect remains and equals

$$\left(\frac{\Delta\nu}{\nu}\right)_d = -\frac{v^2}{2c^2} = \frac{-\varkappa M_{\delta}}{2c^2(r_{\delta} + h)} = -3.5\cdot10^{-10}\left[1 - \frac{h}{r_{\delta}}\right]$$

where, for simplicity, the orbit is considered to be circular. When $h = 800$ km, $\left(\frac{\Delta\nu}{\nu}\right)_d = -3\cdot10^{-10}$, i.e., the quadratic Doppler effect is already four times greater than the gravitational frequency shift. For this reason, and also for a number of other reasons, the difficulties standing in the way of the experiment under discussion should certainly not be underestimated (in this respect, remote satellites are more favorable).

Another effect of the general theory of relativity which can be confirmed by using satellites is the rotation of the perihelion of the orbit. This rotation for earth satellites (in angular seconds per century) equals, according to the well-known Einstein formula:

$$\Psi = \frac{5\pi^2 a^2 Y}{24 c^2 T^3 (1 - e^2)} = 8.35 \cdot 10^{-19} \frac{a^2}{T^3 (1 - e^2)} = \frac{1.74 \cdot 10^{25}}{a^{5/2} (1 - e^2)} \qquad (2)$$

where $Y = 365.25$ is the number of days per year, a is the major semi-axis of the orbit in centimeters, e is the orbit eccentricity and T is the period of satellite rotation in days (certainly, only the last expression refers especially to the earth).

Not taken into account in (2) is the effect of the satellite, exerted by the sun and equal to 7.6'' per century, on the perihelion motion. As already stressed in [6], the effect (2) can be very large for earth satellites. Thus, it attains $\sim 1500''$ per century for satellites close to the earth while $\Psi = 43''$ per century for Mercury. A more detailed analysis [6,1] shows that from the point of view of the possibility of measuring the effect, observations on the satellite throughout a year can be more advantageous than observation of Mercury over a century. Also not excluded is that the use of radio methods to determine the perihelion shift leads to more favorable results. Hence, the final possibility is opened up of detecting the influence of the rotation of the earth on the perihelion shift of the satellite and also on the rotation of the nodes of its orbit.

In a coordinate system, Galilean at infinity, the field of a rotating body has a non-zero component of the $g_{o\alpha}$ $(\alpha = 1,2,3)$ type. Hence, in the case of a weak field far from the body (see [7], § 100):

$$\vec{g} = -\frac{2\varkappa}{c^3 \vec{r}^3} [\vec{I}\vec{r}] \; ; \quad \vec{I} = \int [\vec{r}', \mu\vec{v}] \, dV \qquad (3)$$

where $g_\alpha = -\frac{g_{oa}}{g_{oo}} \cong g_{o\alpha}$, μ is the density of a mass at the point $\vec{r}'$ moving with velocity $\vec{v}$, and $\vec{r}$ is the distance from the center of the body to the observation point. Formula (3) is applicable everywhere (beyond the sphere) for a sphere and for $\mu = \text{const}$: $I = \frac{2}{5} M r_o^2 \omega$, where M is the mass of the sphere,

r_o is its radius and ω is the angular velocity. It was shown in [8] that the field (3) leads to additional rotation of the satellite (planet) perihelion by an angle (in angular seconds per century):

$$(4) \qquad \Psi_B = - \frac{\pi^2 r_o^2 \Upsilon}{9c^2 \tau T^2 (1 - e^2)^{\frac{3}{2}}} ; \qquad \Delta = \frac{|\Psi_B|}{\Psi} = \frac{8}{15}\left(\frac{r_o}{a}\right)^2 \frac{T}{\tau(1 - e^2)^{\frac{1}{2}}}$$

where τ is the period of revolution of central sphere created by the field (in days). It is assumed in (4), for simplicity, that the orbit plane coincides with the equator of the rotating sphere and the rotation of the sphere and the satellite proceeds in the same direction; in the general case, the factor $\left[1 - 3\sin^2\left(\frac{i}{2}\right)\right]$ appears in (4), according to [8], where i is the angle between the equatorial plane and the orbit plane. The angle of rotation of the node is one-half the angle of (4) and of opposite sign. In order to obtain the total effect, the perihelion rotation (4) must simply be added algebraically to the rotation (2).

In the case of Mercury ($a = 5.8\cdot10^{12}$, $T = 88$ days, $r_o = r_\odot = 6.96\cdot10^{10}$ and $\tau = \tau_\odot \simeq 28$ days) $\Delta \cong 2.5\cdot10^{-4}$ and $\Psi_B = -0.01''$, while the admissible accuracy in measuring the rotation of the Mercury perihelion is of the order of $1''$ (see [1]). The picture is different for satellites close to the earth. Thus, for $h = 400$ km, $T \cong 1.54$ hours: $\Delta \simeq 3\cdot10^{-2}$ and $\Psi_B = -43''$ per century, i.e., the relativistic effect related to the rotation of the earth is exactly the same as the whole relativistic effect in the Mercury case. It seems to us that a discussion of the possibility of detecting the relativistic 'rotation effect' should attract attention.

P. N. Lebedev Phys. Inst. October, 1955

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